Finite Math Helper

Dr. Del's Crib Sheet

Linear Models

M 117

Review the Definitions and Formulas
Quick and Easy
Utilized in Triad Math's Online
Finite Math Helper Program
You are invited to visit:
www.FiniteMathHelp.com
for the full story!

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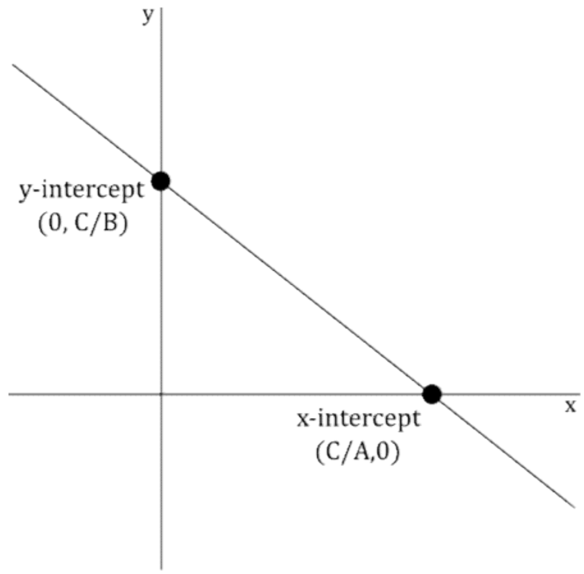
Linear Equation – Straight Line Slope

$$Ax + By = C$$

$$y = -\frac{A}{B}x + \frac{C}{B}$$



$$Slope = -\frac{A}{B} = m$$

$$\frac{C/_{B} - 0}{0 - C/_{A}} = \frac{C/_{B}}{-C/_{A}} = -\frac{A}{B}$$

Linear Equation – Straight Line Slope

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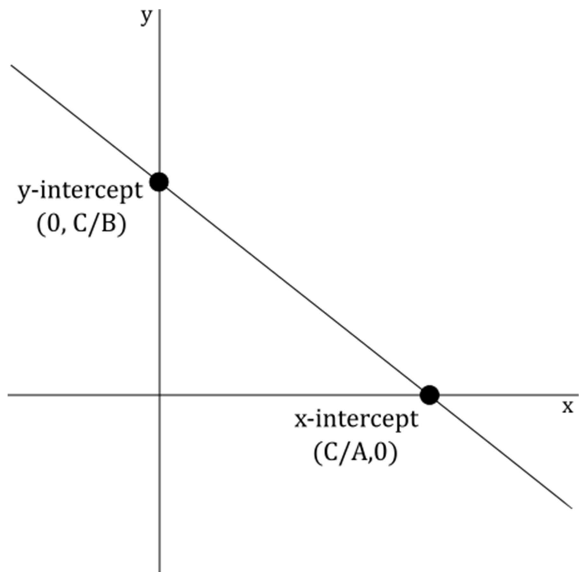
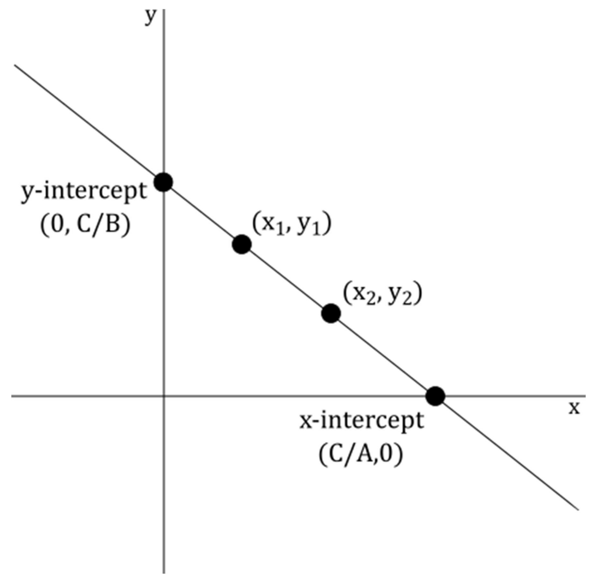


$$Slope = -\frac{A}{B} = m$$

$$\frac{C/_B - 0}{0 - C/_A} = \frac{C/_B}{-C/_A} = -\frac{A}{B}$$

<u>Point – Slope Linear Equation – Straight Line</u>



Slope: $m = \frac{(y_1 - y_2)}{(x_1 - x_2)} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = -\frac{A}{B}$

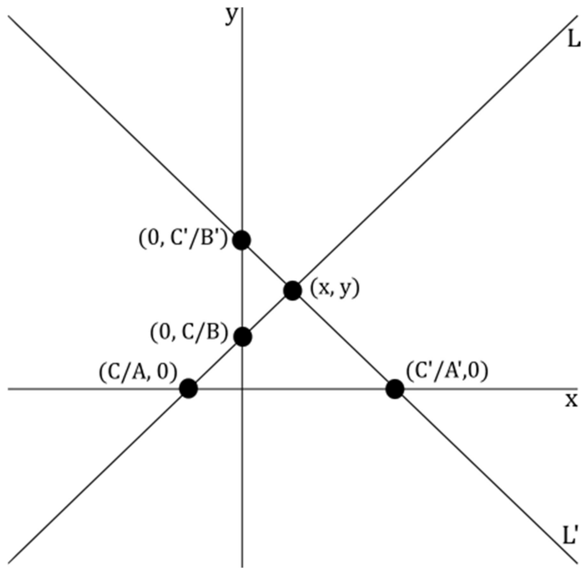
$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

$$y = \frac{(y_1 - y_2)}{(x_1 - x_2)}(x - x_2) + y_2$$

<u>Two Linear Equations – Solutions</u>

L: $\boldsymbol{Ax + By = C}$

L': $\boldsymbol{A'x + B'y = C'}$



$$\boldsymbol{B'Ax + B'By = B'C} \qquad \boldsymbol{A'Ax + A'By = A'C}$$

$$\boldsymbol{BA'x + BB'y = BC'_1} \qquad \boldsymbol{AA'x + AB'y = AC'}$$

$$\boldsymbol{x = \frac{B'C - BC'}{B'A - BA'}} \qquad \boldsymbol{y = \frac{AC' - A'C}{AB' - A'B}}$$

Two Linear Equations – Solutions – cont.

Substitutions

$$\boldsymbol{Ax + By = C} \qquad \boldsymbol{x = [C - By]/A}$$

$$\boldsymbol{B_1 y = C_1} \qquad \boldsymbol{y = C_1/B_1}$$

$$\boldsymbol{x = [C - B(C_1/B_1)]/A}$$

$$\boldsymbol{A_{11}x_1 + A_{12}x_2 = C_1} \qquad \boldsymbol{x_1 = [C_1 - A_{12}(C_1/A_{22})]/A_{11}}$$

$$\boldsymbol{A_{22}x_2 = C_2} \qquad \boldsymbol{x_2 = C_2/A_{22}}$$

Example:

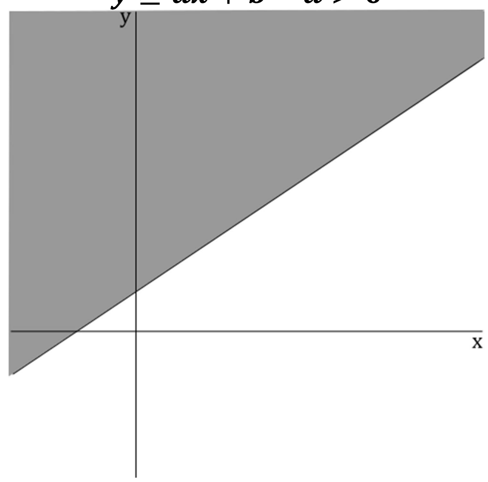
$$\boldsymbol{3x + 4y = 8} \qquad \boldsymbol{-5x + 2y = 9}$$

$$\boldsymbol{5y = 7} \quad \boldsymbol{y = \frac{7}{5}} \qquad \boldsymbol{3x = 7} \quad \boldsymbol{x = \frac{7}{3}}$$

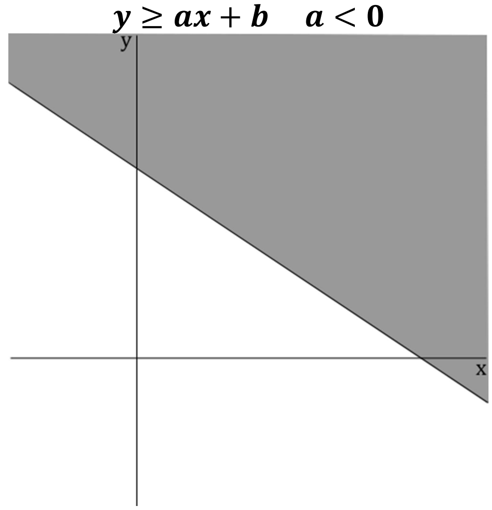
$$\boldsymbol{x = \left[8 - 4\left(\frac{7}{5}\right)\right]/3} \qquad \boldsymbol{y = \left[9 - (-5)\left(\frac{7}{3}\right)\right]/2}$$

Constraining Functions

$$y \geq ax + b \quad a > 0$$

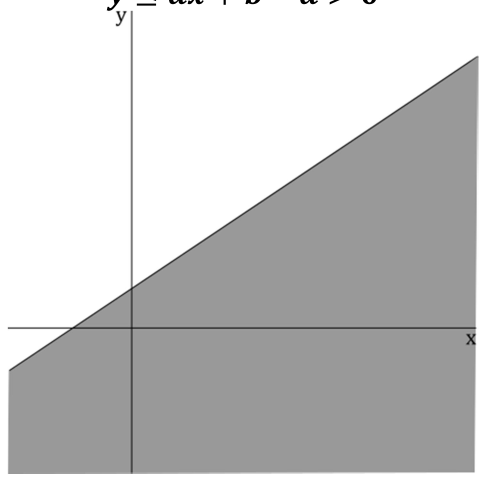


$$y \geq ax + b \quad a < 0$$

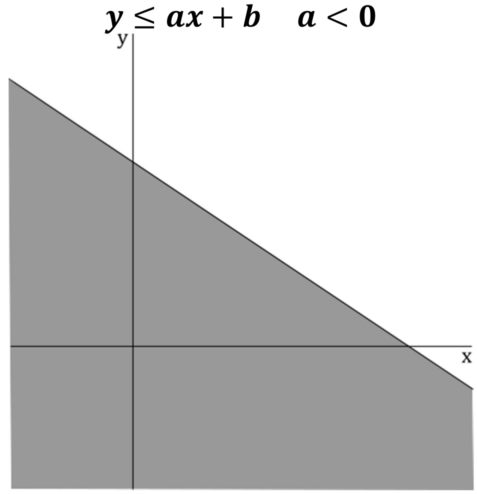


Constraining Functions – cont.

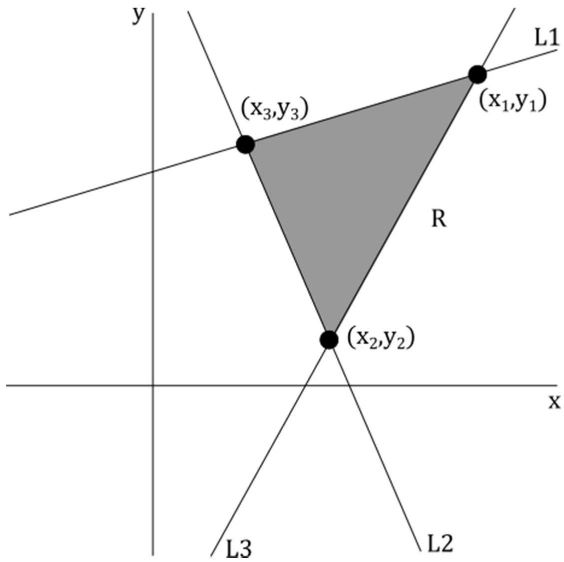
$$y \leq ax + b \quad a > 0$$



$$y \leq ax + b \quad a < 0$$



Linear Programming

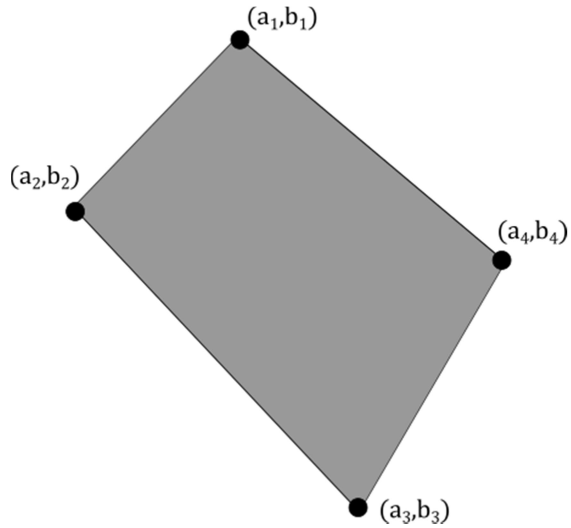


Max & Min $\quad P_1x + P_2y$

$(x_1, y_1) \quad P_1x_1 + P_2y_1$
$(x_2, y_2) \quad P_1x_2 + P_2y_2$
$(x_3, y_3) \quad P_1x_3 + P_2y_3$

L_1, L_1, L_1 are lines of constraint
Define the region to find the max or min of R
The end points $(x_1, y_1) \quad i = 1, 2, 3$

<u>Linear Programming – cont.</u>



$$F(x, y) = Ax + By$$

Evaluate $F(a_1, b_1), F(a_2, b_2), F(a_3, b_3), F(a_4, b_4)$

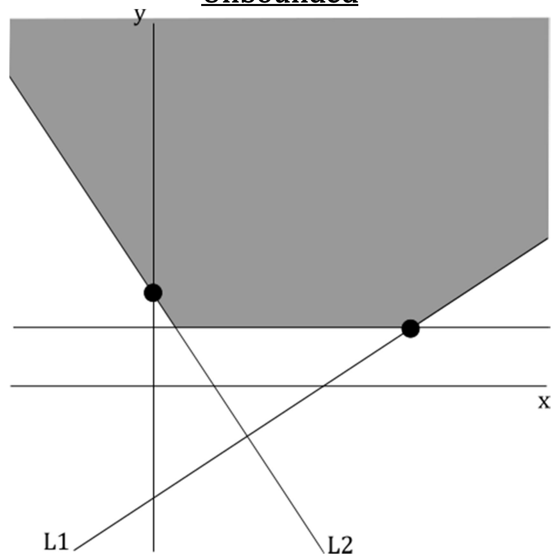
The largest value is the Max.

The smallest value is the Min.

Corner Points

Evaluate objective function at common points to determine maximum or minimum values.

Unbounded



$Ax + By$ Max is unbounded if $A > 0$ and x is unbounded

Min is unbounded if $A < 0$ and x is unbounded

$Ax + By$ Max is unbounded if $B > 0$ and y is unbounded

Min is unbounded if $B < 0$ and y is unbounded

Linear Programming

P_1, P_2 are Products
C_1, C_2, C_3 are Components
T_1, T_2, T_3 are Totals of C_1, C_2, C_3 with R results

		C_1	C_2	C_3	R	
P_1	x	a_1	a_2	a_3	P_1	$a_i \geq 0$
P_2	y	b_1	b_2	b_3	P_2	$b_i \geq 0$
		T_1	T_2	T_3		

$L_1 \quad a_1x + b_1y \leq T_1 \qquad R = P_1x + P_2y$

$L_2 \quad a_2x + b_2y \leq T_2$

$L_3 \quad a_3x + b_3y \leq T_3$

2x2 Matrix Solutions

$$\text{Rows}\quad \overset{\text{Columns}}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} = [a_{ij}] \text{ where } a_{ij}$$

row col

Add

$$A + B = C$$

$$[a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}] = c_{ij}$$

Multiply

$$A \times B = C$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{11} + a_{22}b_{22} \end{bmatrix}$$

$$= [C_{ij}] \text{ where } C_{ij} = a_{i1}b_{j1} + a_{i2}b_{j2}$$

$$Row\ i\ \times Col\ j$$

2x2 Matrix Solutions-cont.

Identity

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \boldsymbol{I} = \textbf{Identity Matrix}$$

Inverse

$$\begin{bmatrix} & \end{bmatrix} \times \begin{bmatrix} & \end{bmatrix} = \boldsymbol{I}$$

A B

$$\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{I} \qquad \boldsymbol{B} = \boldsymbol{A}^{-1}$$

$$\boldsymbol{A} \times \boldsymbol{A}^{-1} = \boldsymbol{A}^{-1} \times \boldsymbol{A} = \boldsymbol{I}$$

$$\begin{bmatrix} \boldsymbol{b}_{11} & \boldsymbol{b}_{12} \\ \boldsymbol{b}_{21} & \boldsymbol{b}_{22} \end{bmatrix} = \boldsymbol{A}^{-1}$$

Inverse Matrix via Gaussian Elimination

$$\left[\begin{array}{cc|cc} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}\right]$$

.
. Gaussian Elimination Rules
.

$$\begin{bmatrix} 1 & 0 & b_{11} & b_{12} \\ 0 & 1 & b_{21} & b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Matrix Rules

Scalar Multiplier: $\boldsymbol{A} \times [\boldsymbol{a_{ij}}] = [\boldsymbol{A}\boldsymbol{a_{ij}}]$

Addition: $\underset{\boldsymbol{n \times m}}{[\boldsymbol{a_{ij}}]} \pm \underset{\boldsymbol{n \times m}}{[\boldsymbol{b_{ij}}]} = \underset{\boldsymbol{n \times m}}{[\boldsymbol{a_{ij}} \pm \boldsymbol{b_{ij}}]}$

$$\boldsymbol{m\ columns}$$

$$\boldsymbol{n\ rows} \quad \boldsymbol{A} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \dots & \boldsymbol{a_{1m}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & \dots & \boldsymbol{a_{2m}} \\ \dots & \dots & \dots & \dots \\ \boldsymbol{a_{n1}} & \boldsymbol{a_{n2}} & \dots & \boldsymbol{a_{nm}} \end{bmatrix} = [\boldsymbol{a_{ij}}]$$

$\boldsymbol{n \times m}$ matrix

Matrix Rules-cont.

Multiplication: $\underset{n \times m}{[a_{ij}]} \times \underset{m \times k}{[b_{ij}]} = \underset{n \times k}{[c_{ij}]}$

$$C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj}$$

Identity Matrix:

$$I = [a_{ij}] \quad a_{ii} = 1, \quad a_{ij} = 0 \quad i \neq j$$

$$A\ n \times n \quad A^{-1}\ n \times n \quad A^{-1} \times A = I$$

$$A \times B \neq B \times A \text{ in general}$$

Matrix Multiply

$$n \text{ rows} \quad \overset{m\ columns}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}} \quad ith \text{ row}$$

$$m \text{ rows} \quad \overset{k\ columns}{\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1k} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{jk} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{n1} & \cdots & b_{mj} & \cdots & b_{mk} \end{bmatrix}}$$

jth col

$$ith \text{ row} \quad i \overset{jth\ column}{\underset{n \times k}{\begin{bmatrix} & & \\ & C_{ij} & \\ & & \end{bmatrix}}}$$

$$C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj}$$

$$[a_{i1}, \dots, a_{im}] \begin{bmatrix} b_{1j} \\ \cdots \\ b_{mj} \end{bmatrix} =$$
$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj} = C_{ij}$$

Inverse Matrix calculation

$$\begin{matrix} & A & & & I & \\ a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{matrix}$$

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Gaussian . Elimination

.

$$\overset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \quad \overset{B}{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}}$$

$$A^{-1} = B$$

Gaussian Elimination Rules

You may substitute a new row by:

1. Multiply a row by a scalar
2. Add two rows
3. Exchange rows

Two Linear Equations – Common Solutions

	Col1		*Col2*			x_1	x_2	
Row 1	$A_{11}x_1$	+	$A_{12}x_2$	=	b_1	A_{11}	A_{12}	b_1
Row 2	$A_{21}x_1$	+	$A_{22}x_2$	=	b_2	A_{21}	A_{22}	b_2

$$x_1 = \frac{A_{22}b_1 - A_{12}b_2}{A_{22}A_{11} - A_{12}A_{21}} \quad x_2 = \frac{A_{11}b_2 - A_{21}b_1}{A_{11}A_{22} - A_{21}A_{12}}$$

Two Linear Equations – Gaussian Elimination

Gaussian Elimination:

1. Exchange rows
2. Multiply a row by a scalar (number)
3. Add rows and exchange

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \overset{\textbf{scalar}}{\rightarrow} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \begin{matrix} x_1 = c_1 \\ x_2 = c_2 \end{matrix}$$

Two Linear Equations – Substitution

$$\begin{matrix} \boldsymbol{x_1} & \boldsymbol{x_2} & \\ \end{matrix}$$
$$\left[\begin{array}{cc|c} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{c_1} \\ \boldsymbol{0} & \boldsymbol{a_{22}} & \boldsymbol{c_2} \end{array}\right] \quad \boldsymbol{a_{21} = 0}$$

$$\boldsymbol{x_2 = \frac{c_2}{a_{22}}}, \qquad \boldsymbol{x_1 = (c_1 - a_{12}x_2)/a_{11}}$$

$$\boldsymbol{a_{11}x_1 + a_{12}x_2 = c_1}$$
$$\boldsymbol{x_1 = c_1 - \ a_{12}x_2/a_{11}}$$

Identity

Gaussian Elimination:

$$\begin{matrix} x_1 & x_2 & x_3 & & \\ \end{matrix}$$
$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & 0a_{32} & a_{33} & b_3 \end{array}\right] \quad \begin{matrix} x_1 = c_1 \\ x_2 = c_2 \\ x_3 = c_3 \end{matrix}$$

or

Substitution:

$$\begin{matrix} x_1 & x_2 & x_3 & \\ \end{matrix}$$
$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ & a_{21} & a_{23} & c_2 \\ & & a_{33} & c_3 \end{array}\right]$$

$$a_{33}x_3 = c_3$$

$$x_3 = c_3/a_{33}$$

$$x_2 = [c_2 - a_{23}x_3]/a_{21}$$

$$x_1 = [c_1 - a_{12}x_2 - a_{13}x_3]/a_{11}$$

or

$$A^{-1} - \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Leontief Linear Economic Model

$\boldsymbol{A\ n\ x\ n}$ Technology Matrix

$\boldsymbol{X\ n\ x\ 1}$ Production Schedule Col Matrix

$\boldsymbol{D\ n\ x\ 1}$ Demand Vector Col Matrix

$$\boldsymbol{X = AX + D}$$
$$\boldsymbol{(1 - A)X = D}$$
$$\boldsymbol{X = (I - A)^{-1}D}$$

$$\boldsymbol{x_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + d_i}$$
$$\boldsymbol{x_i = \#\ \text{of goods labeled}\ i}$$
$$\boldsymbol{a_{ij} = \text{fraction of}\ x_i\ \text{goods to make}\ x_j\ \text{goods}}$$
$$\boldsymbol{a_{ij} \geq 0, I - A\ \text{invertible}}$$

$$\boldsymbol{A = [a_{ij}] \quad X = [x_i] \quad D = [xd_i]}$$

$$\boldsymbol{(I - A)^{-1}\ \text{all non} - \text{negative entries}}$$

Given A and D, compute X

or given A and X, compute D

Finite Math Helper

Linear Models – Math 117

The notebook contains the various definitions and formulas utilized in Chapter Five through Seven of the book, Finite Math, by Thompson, Maki, and McKinley used in M117 and the second half of M 118 at Indiana University.

This notebook was created by Dr. Craig Hane when he studied the course to create a set of videos to help students succeed in the course. Indeed, he says he wishes he had had such a notebook when he first studied the course to help his students. Big Time Saver!

This notebook IS NOT a textbook and it does not teach the student anything. It is just a convenient reference notebook to save time looking up formulas and definitions when solving Finite Math problems.

If you want help in your Finite Math course, the best help is, of course, a good tutor. Obviously, that might be expensive and logistically difficult.

Another resource that could save you much time and frustration is the Online Finite Math Help Program that was created by Triad Math, Inc., consisting of over 100 online videos and a forum. For details visit:

www.FiniteMathHelp.com