Finite Math Helper

Dr. Del's Crib Sheet

Probability Models

M 116

Review the Definitions and Formulas

Quick and Easy

Utilized in Triad Math's OnLine

Finite Math Helper Program

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for the Full Story!

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Finite Math "Probability" Helper

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Finite Math Probability Helper

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Set Theory: Basic Definitions

Symbol	Definition
A, B, S, U	Sets labeled A, B, S, U
$x \in A$	$\{x \mid x$ is a member of set $A\}$
φ	Empty Set - No Elements
U	Universal Set - All Elements
$A \subset B$	$x \varepsilon A$ implies $x \varepsilon B$

We say: "A is a subset of B"

$A \cup B$ $\{x \mid x \varepsilon A$ OR $x \varepsilon B\}$

We say: "A Union B"

$A \cap B$ $\{x \mid x \varepsilon A$ AND $x \varepsilon B\}$

We say: "A intersect B"

If U is Universal set and $A \subset U$ we define

$A' = \{x \mid x \notin A$ and $x \in U\}$

We say: A' is the Complement of A

$\Phi' = U$ and $U' = \varphi$

DeMorgan's Laws

$(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

Set Theory More Definitions

Symbol **Definitions**

n(A) Number of elements in A

AxB $\{(x,y) \mid x \in A \text{ and } y \in B\}$

We say: "The Cross Product of A and B"

Fact: $n(A \times B) = n(A) \times n(B)$

$A \cap B = \phi$ A and B are Disjoint

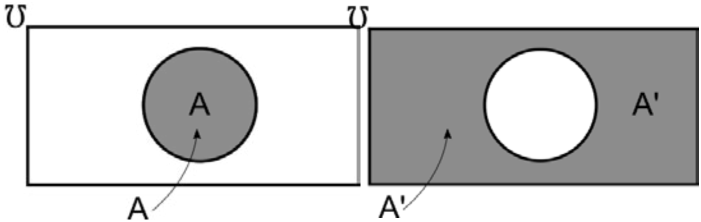
If $E \subset U$, $E' = \{x \mid x \notin E \text{ and } x \in U\}$

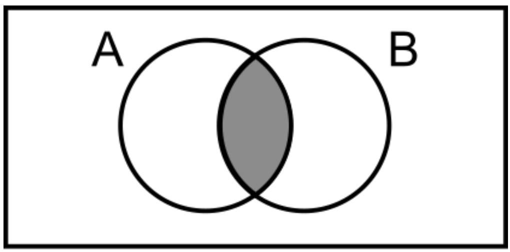
$E \cap E' = \phi$ and $E \cup E' = U$ (A Partition)

$P = \{A_1, A_2, \ldots A_n\}$ is a Partition of U if:

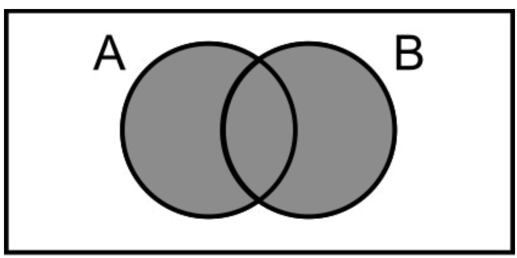
1) $A_1 \cup A_2 \cup \ldots \cup A_n = U$
2) $A_i \cap A_j = 0$ if $i \neq j$
3) $n(U) = n(A_1) + n(A_2) + \ldots + n(A_n)$

Venn Diagrams





$A \cap B$

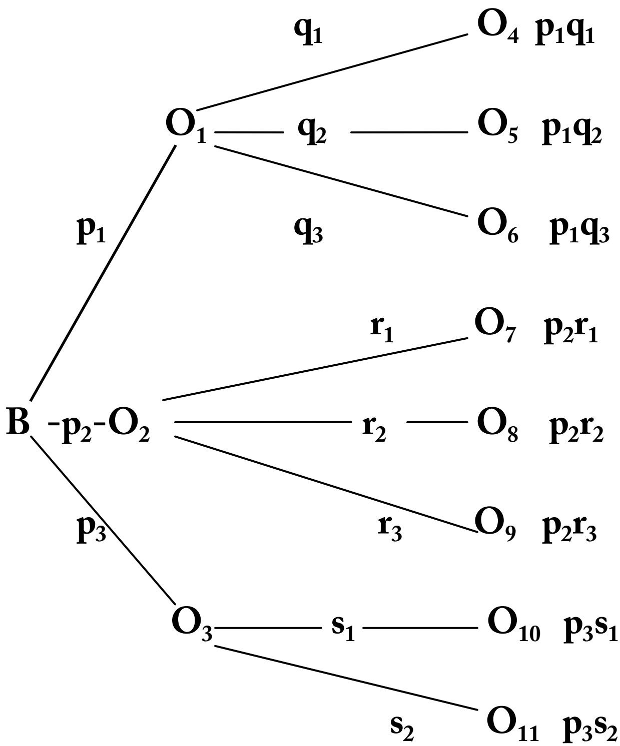


$A \cup B$

See the Online Lessons and Finite Math book for many examples of Venn Diagrams in more complex situations including Probability

TREES: Overview Definition

A Tree is a set of sequence of Outcomes arrayed in Branches.

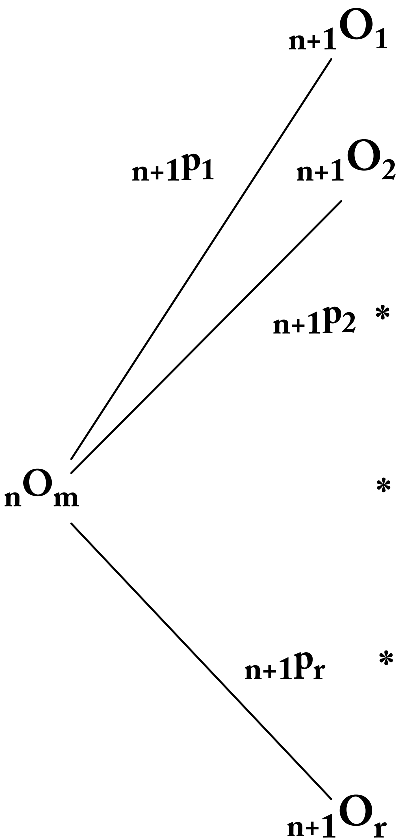


$$p_1+p_2+p_3 = q_1+q_2+q_3 = r_1+r_2+r_3 = s_1+s_1 =$$

$$p_1q_1+p_1q_2+p_1q_3+p_2r_1+p_2r_2+p_2r_3+p_3s_1+p_3s_2 = 1$$

Trees: Stages n to n + 1

Stage n **Stage n + 1**



$${}_{n+1}p_1 + {}_{n+1}p_2 + \ldots {}_{n+1}p_r = 1$$

Trees: n Sequential Stages

P_{01} P_{12} $P_{(n-1)n}$

B — O_1 — O_2 - - - O_{n-1} — O_n

Probability of all n Outputs, O_1 . . . O_n

in Sequence is Product of Probabilities

$$P_{01} \times P_{12} \times \cdots \times P_{(n-1)n}$$

Remember: In a Tree these will all

Add Up to 1.

Counting

The number of different ways to arrange n unique objects is called n Factorial, n!

$$n! = n \times (n-1) \times \ldots \times 2 \times 1$$

On the TI30Xa it is n 2nd 3

The number of different ways to arrange r objects chosen from n objects is called the Permutation of r objects chosen from n objects.

$$P(n,r) = n \times (n-1) \times \ldots (n-r+1) = n!/(n-r)!$$

On the TI30Xa it is: n 2nd 8 r =

The number of different ways to arrange r objects chosen from n objects when the order of the arrangement does not matter is called the Combination of r objects chosen from n objects

$$C(n,r) = P(n,r)/r! = n!/(n-r)!r!$$

On the TI30Xa it is: n 2nd 9 r =

Note: $C(n,r) = C(n, n-r)$ See Pascal's Triangle in Finite Math pp 73-4 for manual method.

Probability: Basic Definition

Let set S be a Sample Space with an

Event, E, a subset of S. $E \subset S$

$E' = \{x \mid x \notin E, x \in S\}$

$E \cap E' = \varphi$ **and** $E \cup E' = S$

n(E) and n(S) and n(E') are given.

E and E' are a Partition of S

$n(E) + n(E') = n(S)$

<u>Probability Event E</u> occurred is:

$$Pr[E] = n(E)/n(S)$$

$$Pr[E] + Pr[E'] = 1 = Pr[S]$$

Probability: Partitions

Let $S = \{x_1, x_2, \ldots, x_n\}$ be a Sample Space where each outcome x_i, is equally probable: $Pr[x_i] = 1/n$ Let $E_1, E_2, \ldots, E_r$, be a <u>Partition of S</u> of disjoint subsets of S each containing one or more x_i's.

$E_1 \cup \ldots \cup E_r = S$ and $E_i \cap E_j = \phi \quad i \neq j$

$$Pr[E_i] = n(E_i)/n(S) \quad i = 1, \ldots, r$$

$$Pr[E_1] + \ldots + Pr[E_r] = 1$$

Example: $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$E_1 = \{1,4,8\}$, $E_2 = \{2\}$, $E_3 = \{3,5,6,7\}$

$Pr[E_1] = n(E_1)/n(S) = 3/8$

$Pr[E_2] = 1/8$; $[Pr[E_3] = 4/8 = ½$

Probability: Experiments

Suppose you conduct an experiment

with n possible outcomes $O_1, \ldots, O_n$

$Pr[Oi] = w_i \quad 0 \le w_i \le 1 \quad i = 1, \ldots, n$

$$w_1 + \ldots + w_n = 1$$

The Sample Space is now the set of Outcomes

$\{ O_1, \ldots, O_n\} = S$

Event E is a subset of outcomes of S.

$$E = \{O_a, O_b, \ldots, O_r\} \subset S$$

$Pr[E] = Pr[O_a] + Pr[O_b] + \ldots + Pr[O_r]$

$$= w_a + w_b + \ldots + w_r$$

Probability: Sequential Experiments

Suppose you perform a Sequence of Experiments, $O_1, O_2, \ldots, O_n$

The Probability of this Sequence occurring is defined to be:

$$Pr[O_1]xPr[O_2]x \ldots xPr[O_n]$$

We see this in Trees where one Branch is this sequence of Outcomes and the Probability of this Branch occurring is this.

The Sum of All the Branches will be 1.

Probability: Conditional Pr[A/B]

Let S be a Sample Space with two Event Subspaces A and B. $A \subset S$ and $B \subset S$ with

$Pr[A] > 0$ and $Pr[B] > 0$

Suppose we are given that B has occurred, then what is the probability that A has occurred too? This is called the Conditional Probability that A occurs given that B has occurred: Pr[A/B]

The Definition is:

$$Pr[A/B] = Pr[A \cap B]/Pr[B]$$

Note: If $A \cap B = \phi$, then $Pr[A/B] = 0$

If $B \subset A$, then $Pr[A/B] = 1$

Probability: $Pr[A/B]$ and $Pr[B/A]$

$Pr[A/B] = Pr[A \cap B]/Pr[B]$ by definition

$Pr[B/A] = Pr[A \cap B]/Pr[A]$ by definition

$Pr[A \cap B] = Pr[A/B] \times Pr[B] = Pr[B/A] \times Pr[A]$

Thus with a little simple Algebra we get:

I $\quad Pr[A/B] = Pr[B/A] \times Pr[A]/Pr[B]$

II $\quad Pr[B/A] = Pr[A/B] \times Pr[B]/Pr[A]$

So, If you know Pr[A], Pr[B], and one of the Conditional Probabilities you can easily calculate the other Conditional Probability using I or II.

See Bayes Theorem for a generalization of this with a Partition of S.

Probability: Bayes Formula

Suppose $S_1, S_2, \ldots, S_n$ is a Partition of a

Sample Space, S

Suppose $Pr[S_i] > 0$ for all $i = 1, \ldots, n$

Suppose A is an Event with $Pr[A] > 0$

Suppose $Pr[A/S_i] > 0$ and $Pr[Si] > 0$ are known for all i. THEN For any i,

$$Pr[S_i/A] = \frac{Pr[A/S_i]\, Pr[S_i]}{Pr[A/S_1]\, Pr[S_1] + \ldots + Pr[A/S_n]\, Pr[S_n]}$$

Bernoulli: Trial and Process Definitions

A <u>Bernoulli Trial</u> is an Experiment with two possible outcomes, s and f .

The Probability of s is $P(s) = p, \quad 0 \le p \le 1$

The Probability of f is $P(f) = q = 1 - p$

[You can think of s and f as "success" and "failure" if you like. They are symmetrical and, of course, $p = 1 - q$]

A <u>Bernoulli Process</u> is a Sequence of Repetitions of a Bernoulli Trial.

Each Trial has the $P(s) = p$ and $P(f) = q = 1 - p$

Bernoulllli: Process Probabilities

Suppose you have a Bernoulli Process consisting of n Bernoulli Trials with the same probabilities.

$$P(s) = p \text{ and } P(f) = q = 1 - p$$

What is the Probability of r successes out of these n Trials? P(Success, n, r)

$$P(\text{Success},n,r) = C(n,r) x p^{r} x (1-p)^{n-r}$$

$$\text{Where } C(n,r) = P(n,r)/r! = n!/(n-r)!r!$$

What is the Probability of (n – r) failures out of these n Trials? P(Failure, n, n-r)

$$P(\text{Failure}, n, n-r) = C(n, n-r) x (1-p)^{n-r} x p^{r}$$

Note: $C(n,r) = C(n,n-r)$

$$P(\text{Success}, n, r) = P(\text{Failure}, n, n-r)$$

Bernoulli: Calculating a Process

$P(\text{Success}, n, r) = C(n,r) \times p^{r} \times (1-p)^{n-r}$

How should one calculate this?

One can look it up in a Table like the one on Page A-15 in the book Finite Mathematics.

Or one can use any Scientific Calculator like the TI30Xa (less than $10). Review the Counting Information on Page 10. You will use the Keys

y^{x} , nCr , STO , RCL , X and =

(Enter Number, Press Key)

1. (p, y^{x}) 2. (r,=) 3. STO 1 4. (1-p, y^{x})

5. (n-r, =) 6. STO 2 7. (n, 2nd 8) 8. (r,=) 9. X

10. RCL 1 11. X 12. RCL 2 13. = Answer

Random Variable: Density Function

Let S be a Sample Space of n Outcomes.

$$S = \{O_1, O_2, \ldots O_n\}$$

A Random Variable, X, is an assignment of a unique real number (+ or -), x_i, to each O_i in S.

$$O_i < -- > x_i$$

In math terms X is a function $X: S \rightarrow R$ where R is the set of Real Numbers.

The <u>Density Function</u> of a Random Variable, X, is an assignment of a probability p_i to each x_i where $p_1 + p_2 + \ldots + p_n = 1$ and $0 \leq p_i \leq 1$

So: $P[O_i] = p_i = P[x_i]$

Random Variable: Expected Value, Variance, Standard Deviation

Let $X = \{x_1, \ldots, x_n\}$ be a Random Variable

with Density Function, P, where

$$P[x_i] = p_i \text{ for } i = 1, \ldots, n$$

The Expected Value, (Mean Value), of X is

$$E[X] = p_1x_1 + p_2x_2 + \ldots + p_nx_n = \mu$$

The Variance of X is Var[X] =

$$(x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \ldots + (x_n - \mu)^2 p_n$$

The Standard Deviation of X is

$\sigma(X) = \{Var[X]\}^{1/2}$ i.e. Square Root of Var[X]

Binomial Random Variable

Suppose you have a Bernoulli Process (p.18) consisting of n Bernoulli Trials with

$$P(s) = p \text{ and } P(f) = q = 1 - p$$

The Random Variable, X, which assigns the number of Successes to this Bernoulli Process of n Bernoulli Trials is called a:

<u>Binomial Random Variable</u>.

$E[X] = n \times p$ x means "multiply"

$Var[X] = n \times p \times (1-p)$

$\sigma(X) = \{ n \times p \times (1-p)\}^{1/2}$

Finite Math Helper

Probability Models - Math 116

This Notebook contains the various Definitions and Formulas utilized in the first four Chapters of the book, Finite Math, by Thompson, Maki and McKinley that is used in M 116 at Indiana University.

This Notebook was created by Dr. Craig Hane when he studied this course to create a set of videos to help students succeed in the course. Indeed, he says he wishes he had had such a Notebook when he first studied the course to help his students. Big Time Saver!

This Notebook IS NOT a textbook and it does not teach the student anything. It is just a convenient reference Notebook to save time looking up Formulas and Definitions when solving Finite Math problems.

If you want help in your Finite Math course the best help is, of course, a good tutor. Obviously, that might be expensive and logistically difficult.

Another resource that should save you much time and frustration is the Online Finite Math Help Program that was created by Triad Math, Inc,, that consists of many Online Videos and a Forum. See the Back Cover.